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Naturally Light Higgs Doublets in the Supersymmetric Grand Unified Theories with Dynamical Symmetry Breaking

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Abstract

We construct a supersymmetric grand unified model in which a dynamical symmetry breaking of SU(5)$_{GUT}$ generates large masses for color-triplet Higgs multiplets while keeping SU(2)$_L$-doublet Higgs multiplets naturally light. The vanishing masses for the Higgs doublets are guaranteed by the Peccei-Quinn symmetry. We also show that the Peccei-Quinn symmetry suppresses the dangerous dimension 5 operators for the nucleon decay.

A basic assumption of the grand unified theories (GUT's) [1] is the presence of a large hierarchy between two mass scales $M_{GUT} \sim 10^{16}$GeV and $m_W \sim 10^2$GeV. It is, however, very much unlikely that such a large mass hierarchy survives the quantum correction without any symmetry reason. Supersymmetry (SUSY) is well known symmetry [2] to protect the hierarchy built at the tree level against the radiative corrections. However, SUSY itself is unable to explain the tree-level hierarchy. In fact, one has to fine tune unrelated parameters in the superpotential to realize the required large hierarchy in the minimum SUSY-GUT [3]. Although there have been proposed several mechanisms [4, 5, 6] to build naturally the hierarchy in SUSY-GUT's, each mechanism has its own problem and no convincing solution has been found so far.

The purpose of this letter is to show a new solution to this serious problem in the SUSY-GUT's. We find that a dynamical symmetry breaking at the GUT scale produces large masses for color-triplet Higgs multiplets while $\mathrm{SU}(2)_L$-doublet Higgs multiplets are kept light naturally. Here, the vanishing masses for the Higgs doublets are guaranteed by the Peccei-Quinn (PQ) symmetry [7]. We note that the dangerous dimension 5 ($d = 5$) operators [8] for the nucleon decay are suppressed by this PQ symmetry.

The model is based on a strong hypercolor gauge group $\mathrm{SU}(3)_H$ acting on 7+7 chiral supermultiplets,

$$\begin{aligned} \phi^A_\alpha &= \varphi^A_\alpha + \theta\psi^A_\alpha, \\ \bar{\phi}^\alpha_A &= \bar{\varphi}^\alpha_A + \theta\bar{\psi}^\alpha_A, \\ (\alpha = 1-3,\ &A = 1-7) \end{aligned} \tag{1}$$

which transform as $\mathbf{3} + \mathbf{3}^*$ under the $\mathrm{SU}(3)_H$. At the classical level this theory has a global flavor symmetry,

$$G_0 = SU(7)_1 \times SU(7)_2 \times U(1)_1 \times U(1)_2 \times U(1)_R, \tag{2}$$

where $\mathrm{U}(1)_R$ corresponds to the phase rotation of the $\mathrm{SU}(3)_H$ gauge fermions (R symmetry). The axial $\mathrm{U}(1)_A \equiv \mathrm{U}(1)_{1-2}$ and $\mathrm{U}(1)_R$ have hypercolor $\mathrm{SU}(3)_H$ anomalies, with a

linear combination $\mathrm{U}(1)_{\bar{R}}$ of these two symmetries being anomaly free. Thus, the global continuous symmetry at the quantum level is

$$G = SU(7)_1 \times SU(7)_2 \times U(1)_V \times U(1)_{\bar{R}}, \tag{3}$$

where the $\mathrm{U}(1)_V$ is a diagonal subgroup $\mathrm{U}(1)_{1+2}$.

We make a dynamical assumption that the Higgs and confining phases are smoothly connected [9]. This complementarity picture has been observed by lattice calculations [9] in the theories with elementary scalar fields being the fundamental representations of the gauge group. Since the scalar components φ_α^A and $\bar{\varphi}_A^\alpha$ in the present model are all the fundamental **3** and $\mathbf{3}^*$ of $\mathrm{SU}(3)_H$, our dynamical assumption seems quite reasonable [10]. Using this complementarity picture, we first discuss a vacuum (i.e. symmetry breaking) in the Higgs phase and then reinterpret the result in the confining phase of $\mathrm{SU}(3)_H$.

In the Higgs phase, the scalar fields φ_α^A and $\bar{\varphi}_A^\alpha$ have vacuum-expectation values of the form,

$$\langle \varphi_\alpha^A \rangle = \langle \bar{\varphi}_A^\alpha \rangle = v\delta_A^\alpha, \tag{4}$$
$$(\alpha = 1-3).$$

In this vacuum the global symmetry G_0 is spontaneously broken down to

$$G_0 \longrightarrow H_0 = SU(3)_C \times SU(4)_1 \times SU(4)_2 \times U(1)'_1 \times U(1)'_2 \times U(1)'_R. \tag{5}$$

In the confining phase, the symmetry breaking Eq. (5) in the Higgs phase is achieved by the following $\mathrm{SU}(3)_H$ invariant condensation;

$$\langle \phi_\alpha^A \bar{\phi}_B^\alpha \rangle = \begin{cases} v^2 \delta_B^A, & \text{for } A, B = 1-3 \\ 0, & \text{for others} \end{cases} \tag{6}$$

$$\epsilon^{\alpha\beta\gamma}\epsilon_{ABC}\langle \phi_\alpha^A \phi_\beta^B \phi_\gamma^C \rangle = \epsilon_{\alpha\beta\gamma}\epsilon^{ABC}\langle \bar{\phi}_A^\alpha \bar{\phi}_B^\beta \bar{\phi}_C^\gamma \rangle = v^3, \qquad \text{for } A, B, C = 1-3 \tag{7}$$

where $\epsilon_{\alpha\beta\gamma}, \epsilon_{ABC}, \ldots.$ are the third-rank antisymmetric tensors. This condensation is, therefore, a basic assumption in the present analysis.

We now gauge SU(5)$_{GUT}\times$U(1)$_S$ which is a subgroup of the global symmetry G$_0$. The elementary chiral multiplets, ϕ^I_α and $\bar{\phi}^\alpha_I$ ($I = 1-5$), are assumed to transform as **5** and **5*** under the grand unified SU(5)$_{GUT}$ and the multiplets, $\phi^{\ell+5}_\alpha$ and $\bar{\phi}^\alpha_{\ell+5}$ ($\ell = 1, 2$), are singlets of the SU(5)$_{GUT}$.(I, J denote the SU(5)$_{GUT}$ indices which run from 1 to 5 while A, B, C the global SU(7)$_{1,2}$ indices running from 1 to 7.) Charges of the U(1)$_S$ for ϕ^A_α and $\bar{\phi}^\alpha_A$ are chosen as $Q_S(\phi^A_\alpha) = 1$ and $Q_S(\bar{\phi}^\alpha_A) = -1$ for all $A = 1-7$.

The condensation given in Eqs. (6,7) causes the breaking of the flavor gauge symmetry,

$$SU(5)_{GUT} \times U(1)_S \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{8}$$

where U(1)$_Y$ is a linear combination of an U(1) subgroup of SU(5)$_{GUT}$ and the U(1)$_S$.[1] [2] We assume that all quark, lepton and Higgs multiplets have vanishing charges of U(1)$_S$ and hence they belong to the standard representations of SU(5)$_{GUT}$. Then, the coupling constant g_Y of the U(1)$_Y$ gauge multiplet to the ordinary fields is given by

$$g_Y = g_5(1 + \frac{\alpha_5}{15\alpha_{1S}})^{-1/2} \tag{9}$$

[1] The necessity of introducing the extra gauge group U(1)$_S$ comes from that the condensation in Eq. (7) breaks an U(1) subgroup of the SU(5)$_{GUT}$. If one assumes only the condensation in Eq. (6), one has the desired breaking of SU(5)$_{GUT}$, SU(5)$_{GUT}$ $\longrightarrow$ SU(3)$_C\times$SU(2)$_L\times$U(1)$_Y$. If it is the case, one needs not introduce the extra U(1)$_S$ and the unification of three gauge coupling constants, g_C, g_L and g_Y, is an automatic result of the grand unification. To check whether this vacuum in fact exists an intensive study on the dynamics of SUSY confining forces are required, that is, however, beyond the scope of the present paper.

[2] The U(1)$_Y$ gauge multiplet V$_Y$ is given by

$$V_Y = \frac{(\sqrt{15}g_{1S}V_{24} + g_5 V_S)}{\sqrt{15g_{1S}^2 + g_5^2}}$$

where V$_{24}$ and V$_S$ are gauge vector multiplets of the U(1) subgroup of SU(5)$_{GUT}$ and the extra U(1)$_S$, respectively. The gauge multiplet orthogonal to the V$_Y$ receives a large mass from the condensation in Eq. (7). However, in the limit $g_{1S} \to \infty$, the V$_Y$ is dominated by the V$_{24}$. Thus, one may understand that the GUT relation in Eq. (11) holds in this strong coupling limit as shown in the text. As for the definition of g_5 and V$_{24}$ see Ref.[11] for example, and for the coupling constant g_{1S} see below in the text.

where $\alpha_5 \equiv g_5^2/4\pi$ and $\alpha_{1S} \equiv g_{1S}^2/4\pi$ are the gauge coupling constants of the SU(5)$_{GUT}$ and U(1)$_S$ at the GUT scale v, respectively. Here, the coupling constant g_S is defined as

$$L_{int} = \int d^4\theta \left\{ \phi_\alpha^{*A} e^{-g_S V_S} \phi_\alpha^A + \bar{\phi}_A^{*\alpha} e^{g_S V_S} \bar{\phi}_A^\alpha \right\} \tag{10}$$

with V$_S$ being the gauge vector multiplet of the U(1)$_S$.

In the strong coupling limit of the U(1)$_S$, i.e. $\alpha_{1S} \to \infty$, we recover the GUT unification of the three gauge coupling constants [3]

$$g_C = g_L = g_Y = g_5. \tag{11}$$

If one requires this GUT relation by 1% accuracy, one gets a constraint, $\alpha_{1S} \geq 0.2$, for $\alpha_5 \simeq 1/20$ at the GUT scale.[4]

Let us discuss massless bound states in the confining phase, switching off the flavor gauge interactions (i.e. $g_5 = g_{1S} = 0$). Corresponding to the symmetry breaking $G_0 \longrightarrow H_0$ in Eq. (5), 58 mssless composite Nambu-Goldstone(NG) bosons appear. They are scalar components of the massless composite chiral multiplets [12],[5]

$$\begin{aligned} \bar{\Phi}_{i+3}^a = (\phi_\alpha^a \bar{\phi}_{i+3}^\alpha), \qquad & \Phi_a^{i+3} = (\phi_\alpha^{i+3} \bar{\phi}_a^\alpha), \\ \Phi_b^a &= (\phi_\alpha^a \bar{\phi}_b^\alpha), \\ \Phi_0 &= \epsilon_{\alpha\beta\gamma}\epsilon_{abc}(\phi_\alpha^a \phi_\beta^b \phi_\gamma^c - \bar{\phi}_a^\alpha \bar{\phi}_b^\beta \bar{\phi}_c^\gamma), \\ & (a, b, c = 1-3,\ i = 1-4). \end{aligned} \tag{12}$$

[3] The flavor gauge symmetry and its breaking is very similar to those in the flipped SU(5) model [5]. However, in the flipped model one needs a fine tuning between the two gauge coupling constants of SU(5) and U(1) in order to have the GUT relation in Eq. (11).

[4] Since the U(1)$_S$ is not asymptotic free theory, the coupling α_{1S} may blow up below the Planck scale. Thus, it is very interesting to embed the strong gauge groups SU(3)$_H$ and U(1)$_S$ into some larger non-Abelian group together. This possibility will be investigated elsewhere. Another solution to this problem is to consider that the strong U(1)$_S$ has an ultra-violet fixed point.

[5] In the Higgs phase we have the corresponding NG chiral multiplets,

$$\bar{\phi}_{i+3}^\alpha, \;\; \phi_\alpha^{i+3}, \;\; (\phi_\alpha^b + \bar{\phi}_b^\alpha), \quad (\phi_\alpha^\alpha - \bar{\phi}_\alpha^\alpha).$$

Notice that $a, b, c = 1-3$ and $i+3 = 4, 5$ denote the color $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ indices, but $\alpha, \beta, \gamma = 1-3$ the hypercolor $\mathrm{SU}(3)_H$ indices. The Φ^a_b and Φ_0 contain 9 and 1 quasi-NG bosons,[12, 10], respectively. Thus, we have totally 68 massless scalar bosons.

The unbroken group H_0 has two U(1)'s. The new R symmetry $\mathrm{U}(1)'_R$ is free from the hypercolor anomaly. The axial $\mathrm{U}(1)'_A \equiv \mathrm{U}(1)'_{1-2}$ is broken down to a discrete Z_8 by the $\mathrm{SU}(3)_H$ anomaly. Thus, the unbroken symmetry H at the quantum level is

$$H = SU(3)_C \times SU(4)_1 \times SU(4)_2 \times U(1)'_V \times U(1)'_R \times Z_8, \tag{13}$$

where $\mathrm{U}(1)'_V$ is a diagonal subgroup of $\mathrm{U}(1)'_1 \times \mathrm{U}(1)'_2$, i.e.$\mathrm{U}(1)'_V \equiv \mathrm{U}(1)'_{1+2}$.

With the unbroken H in Eq. (13) we have checked that the chiral fermions of the composite NG multiplets in Eq. (12) satisfy all 'tHooft anomaly matching conditions[13]. This also strongly supports our dynamical assumption in Eq. (6,7).

When we switch on the $\mathrm{SU}(5)_{GUT}\times$ $\mathrm{U}(1)_S$ gauge interactions, 12 NG chiral multiplets, Φ^a_{i+3} and $\bar{\Phi}^{i+3}_a$ $(i = 1, 2)$ and one Φ_0 are absorbed to massive vector multiplets of $\mathrm{SU}(5)_{GUT}\times$ $\mathrm{U}(1)_S$. To eliminate unwanted 9 Φ^a_b in the masslss spectrum one may introduce a mass for $\phi^I_\alpha(\mathbf{5})$ and $\bar{\phi}^\alpha_I(\mathbf{5}^*)$, $m_\phi\ \phi^I_\alpha \bar{\phi}^\alpha_I$. However, in this case the condensation $\langle \phi^I_\alpha \rangle = \langle \bar{\phi}^\alpha_I \rangle \neq 0$ breaks the SUSY in the Higgs phase, since it gives a non-vanishing vacuum energy. A simple way to solve this problem is to add a new chiral multiplet Σ^I_J transforming as $(\mathbf{1}+\mathbf{24})$ representation of the $\mathrm{SU}(5)_{GUT}$. Then, we take the following superpotential;

$$W = m_\phi \phi^I_\alpha \bar{\phi}^\alpha_I + \lambda \phi^I_\alpha \Sigma^J_I \bar{\phi}^\alpha_J + \frac{M_\Sigma}{2} Tr(\Sigma^I_J)^2, \qquad (I, J = 1-5). \tag{14}$$

We have a SUSY-invariant vacuum in the Higgs phase,

$$\langle \Sigma^I_J \rangle = \begin{cases} -\frac{m_\phi}{\lambda}\delta^I_J, & \text{for } I, J = 1-3 \\ 0, & \text{for others} \end{cases} \tag{15}$$

and $\langle \phi^I_\alpha \rangle$ and $\langle \bar{\phi}^\alpha_I \rangle$ given in Eq. (4) with

$$v = \frac{1}{\lambda}\sqrt{m_\phi M_\Sigma}. \tag{16}$$

In the confining phase the Yukawa coupling, $\lambda\phi^I_\alpha\Sigma^J_I\bar{\phi}^\alpha_J$, in Eq. (14) induces a mass mixing between the Φ^a_b and Σ^I_J as

$$W_{eff} \sim \lambda v \Sigma^b_a \Phi^a_b. \tag{17}$$

Diagonalization of the mass matrix for Φ and Σ generates the mass for the Φ^a_b,[6]

$$W_{mass} \sim \frac{(\lambda v)^2}{M_\Sigma}\Phi^a_b\Phi^b_a. \tag{18}$$

Composite states remaining in the massless spectrum are now

$$\Phi^{\ell+5}_a \quad \text{and} \quad \bar{\Phi}^a_{\ell+5}, \qquad (\ell = 1, 2) \tag{19}$$

which transform as **3** and **3*** under the color SU(3)$_C$.[7] It should be noted here that there still remains a continuous global symmetry, SU(2)$_1\times$ SU(2)$_2\times$ U(1)$''_V$, which is a subgroup of the SU(4)$_1\times$ SU(4)$_2\times$ U(1)$'_V$ in Eq. (13). The presence of the above two pairs ($\ell = 1, 2$) of massless chiral multiplets is required by this unbroken chiral SU(2)$_1\times$ SU(2)$_2$, where the composite $\Phi^{\ell+5}_a$ and $\bar{\Phi}^a_{\ell+5}$ are (**2**,**1**) and (**1**,**2**) representations, respectively. There is also an axial U(1)$''_A$ under which the original chiral multiplets transform as

$$\begin{aligned} \phi^I_\alpha,\ \bar{\phi}^\alpha_I &\longrightarrow \phi^I_\alpha,\ \bar{\phi}^\alpha_I, \quad (I = 1-5) \\ \phi^{\ell+5}_\alpha, \bar{\phi}^\alpha_{\ell+5} &\longrightarrow e^{i\alpha}\phi^{\ell+5}_\alpha, e^{i\alpha}\bar{\phi}^\alpha_{\ell+5} \quad (\ell = 1, 2). \end{aligned} \tag{20}$$

Although this U(1)$''_A$ is broken down to a discrete Z$_4$ by the hypercolor SU(3)$_H$ anomaly, the Z$_4$ keeps the masslessness of the color-triplet composite supermultiplets in Eq. (19).[8]

[6] In the Higgs phase, the corresponding NG multiplets $\frac{1}{\sqrt{2}}(\phi^a_\alpha + \bar{\phi}^\alpha_a)$ acquire also masses $\sim (\lambda v)^2/M_\Sigma$ through the mixing with Σ^I_J.

[7] If one assume SU(2) as the hypercolor gauge group instead of the SU(3)$_H$, the remaining massless states are SU(2)$_L$-doublets, which may be identified as two pairs of Higgs doublets. In this case, however, one needs nonrenormalizable interactions to have effective Yukawa couplings of these Higgs multiplets to the ordinary quarks and leptons.

[8] If one assumes 6+6 chiral multiplets, ϕ^A_α and $\bar{\phi}^\alpha_A$ ($A = 1-6$), one have a pair of massless states, Φ^6_a

It is now clear that we can easily incorporate the missing partner mechanism [4] in the present model. Let us introduce two pairs of Higgs multiplets $H_I^{(\ell)}$ and $\bar{H}^I_{(\ell)}$ with $\ell = 1, 2$ that are **5** and **5*** of SU(5)$_{GUT}$ and add a possible Yukawa couplings,

$$W = \bar{h}\phi^I_\alpha \bar{\phi}^\alpha_{\ell+5} H_I^{(\ell)} + h\phi^{\ell+5}_\alpha \bar{\phi}^\alpha_I \bar{H}^I_{(\ell)}. \tag{21}$$

These Yukawa interactions preserve the chiral $SU(2)_1 \times SU(2)_2$ symmetry under which the elementary chiral multiplets, $\phi^{\ell+2}_\alpha, \bar{H}^I_{(\ell)}$ and $\bar{\phi}^\alpha_{\ell+5}, H_I^{(\ell)}$ transform as (**2**,**1**) and (**1**,**2**) representations, respectively.

The Yukawa couplings in Eq. (21) give rise to masses for the NG multiplets $\Phi^{\ell+5}_a$, $\bar{\Phi}^a_{\ell+5}$ in Eq. (19) and the elementary $H_a^{(\ell)}, \bar{H}^a_{(\ell)}$ as

$$W_{mass} \sim \bar{h}v\bar{\Phi}^a_{\ell+5} H_a^{(\ell)} + hv\Phi^{\ell+5}_a \bar{H}^a_{(\ell)}. \tag{22}$$

The SU(2)$_L$-doublet Higgs $H_i^{(\ell)}$ and $\bar{H}^i_{(\ell)}$ $(i = 4, 5)$ remain naturally massless, since they do not have partners to form invariant masses and, furthermore, the masslessness of these doublet-Higgs multiplets is guaranteed by the unbroken SU(2)$_1\times$ SU(2)$_2$ symmetry. On the other hand, the color-triplet $H_a^{(\ell)}$ and $\bar{H}^a_{(\ell)}$ have the partners $\bar{\Phi}^a_{\ell+5}$ and $\Phi^{\ell+5}_a$ and hence they can form two pairs of massive scalar multiplets, whose masses are invariant under the SU(2)$_1\times$SU(2)$_2$. It should be stressed that the remaining unbroken symmetry is now SU(3)$_C\times$SU(2)$_L\times$U(1)$_Y\times$SU(2)$_1\times$ SU(2)$_2\times$U(1)$''_V$.[9], with which all the 'tHooft anomaly matching conditions are satisfied. Namely, the global anomalies arising from loops of the elementary fields, ϕ^A_α, $\bar{\phi}^\alpha_A$, $H_I^{(\ell)}$ and $\bar{H}^I_{(\ell)}$, are reproduced by the surviving doublet Higgs, $H_i^{(\ell)}$ and $\bar{H}^i_{(\ell)}$.[10]

and $\bar{\Phi}^a_6$, whose mass seems to be forbidden by an axial U(1)$_A$ symmetry. However, the U(1)$_A$ is broken down to Z$_2$ by the hypercolor anomaly and the mass for these composite multiplets may be induced by the hypercolor instanton effects. This is a main reason why we assume the hypercolor theory with 7+7 chiral multiplets.

[9] The U(1)$'_R$ is explicitely broken by the superpotential in Eq. (14).

[10] When one examines the chiral anomalies at the elementary-field level one should remember that the

The first pair of the SU(2)$_L$-doublet Higgs $H_I^{(1)}$ and $\bar{H}^I_{(1)}$ are assumed to have Yukawa couplings to quark and lepton chiral multiplets $\mathbf{5}^*$ and $\mathbf{10}$,

$$\mathbf{5}^* \cdot \mathbf{10}\bar{H}^I_{(1)} + \mathbf{10} \cdot \mathbf{10} H_I^{(1)}. \tag{23}$$

These Yukawa interactions breaks the chiral SU(2)$_1\times$ SU(2)$_2$, but preserve the maximum subgroup U(1)$_1\times$U(1)$_2$. Important for the present analysis is the axial U(1)$_A$ $\equiv$U(1)$_{1-2}$ under which

$$\begin{aligned} H_I^{(1)},\ \bar{H}^I_{(1)} &\longrightarrow e^{i\beta} H_I^{(1)},\ e^{i\beta}\bar{H}^I_{(1)}, \\ H_I^{(2)},\ \bar{H}^I_{(2)} &\longrightarrow e^{-i\beta} H_I^{(2)},\ e^{-i\beta}\bar{H}^I_{(2)}, \\ \mathbf{5}^*,\ \mathbf{10} &\longrightarrow e^{-(i/2)\beta}\mathbf{5}*,\ e^{-(i/2)\beta}\mathbf{10}. \end{aligned} \tag{24}$$

This is nothing but the Peccei-Quinn (PQ) symmetry[7]. This PQ symmetry has no hypercolor SU(3)$_H$ anomaly, but has a color SU(3)$_C$ anomaly providing a solution to the strong CP problem. Thanks to this U(1)$_A$ the two pairs of Higgs doublets, $H_i^{(\ell)}$ and $\bar{H}^i_{(\ell)}$ $(i=4,5)$ still remain massless.[11]

generator of U(1)$_Y$ is a linear combination of generators of the 24^{th} component of SU(5)$_{GUT}$ and of the U(1)$_S$.

[11] It seems that the mass terms, $H_i^{(1)}\bar{H}^i_{(2)}$ and $H_i^{(2)}\bar{H}^i_{(1)}$, are allowed. However, it is not the case, since there is, in fact, a larger chiral symmetry U(1)$_1\times$U(1)$_2$ as noted in the text. By this chiral symmetry those mass terms are completely forbidden. The PQ U(1)$_A$ is the diagonal subgroup U(1)$_{1-2}$ and another combination U(1)$_{1+2}$ is so-called 5-ness symmetry defined as the following transformations;

$$\begin{aligned} \phi^I_\alpha,\ \ \bar{\phi}^\alpha_I &\longrightarrow \phi^I_\alpha,\ \bar{\phi}^\alpha_I, \\ \phi^6_\alpha \longrightarrow e^{-2i\gamma}\phi^6_\alpha,\quad \bar{\phi}^\alpha_6 &\longrightarrow e^{2i\gamma}\bar{\phi}^\alpha_6, \\ \phi^7_\alpha \longrightarrow e^{2i\gamma}\phi^7_\alpha,\quad \bar{\phi}^\alpha_7 &\longrightarrow e^{-2i\gamma}\bar{\phi}^\alpha_7, \\ H_I^{(1)} \longrightarrow e^{-2i\gamma}H_I^{(1)},\ \ \bar{H}^I_{(1)} &\longrightarrow e^{2i\gamma}\bar{H}^I_{(1)}, \\ H_I^{(2)} \longrightarrow e^{2i\gamma}H_I^{(2)},\ \ \bar{H}^I_{(2)} &\longrightarrow e^{-2i\gamma}\bar{H}^I_{(2)}, \\ \mathbf{5}^* \longrightarrow e^{-3i\gamma}\mathbf{5}^*,\quad \mathbf{10} &\longrightarrow e^{i\gamma}\mathbf{10}. \end{aligned}$$

As for the breaking of the PQ symmetry we assume an asymmetric pair of PQ multiplets[15], ϕ_P and ϕ_Q, which transform as $\phi_P \to e^{2i\beta}\phi_P$ and $\phi_Q \to e^{-6i\beta}\phi_Q$. Since the ϕ_P can couple only to the second pair of Higgs $H_I^{(2)}$ and $\bar{H}^I_{(2)}$, the PQ breaking generates a mass at O(10^{12}) GeV for $H_i^{(2)}$ and $\bar{H}^i_{(2)}$ while the first pair of Higgs doublets $H_i^{(1)}$ and $\bar{H}^i_{(1)}$ with $i = 1, 2$ remain massless[15]. As pointed out in Ref. [14], the unification of three gauge coupling constants can be maintained as soon as the masses of the color-triplet Higgs multiplets are taken as $\sim$O(10^{14}) GeV.

The final comment is on the nucleon decay. In the present model, the dangerous $d = 5$ operators[8], $\mathbf{5}^* \cdot \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{10}$, are forbidden by the PQ symmetry. Even if the mass term, $m_{(2)} H_I^{(2)} \bar{H}^I_{(2)}$, is generated by the PQ breaking, the $d = 5$ operators are still suppressed, since $H_I^{(2)}$ and $\bar{H}^I_{(2)}$ do not have Yukawa couplings to quark and lepton multiplets. Therefore, it is very much unlikely that the $d = 5$ operators give dominant contributions to the nucleon decay.

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